Pongamos :

$K = \frac{1}{2} g^{\alpha\beta} g^{\mu\nu}$

Entonces :

$J^{\mu} = g^{\alpha\beta} \partial \Gamma^{\mu}_{\alpha\beta} - g^{\beta\mu} \partial \Gamma^{\lambda}_{\lambda\beta}$

$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} (\partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\nu\beta} - \partial_\nu g_{\alpha\beta})$

$\Gamma^{\lambda}_{\lambda\beta} = \frac{1}{2} g^{\lambda\nu} (\partial_\beta g_{\nu\lambda} + \partial_\lambda g_{\nu\beta} - \partial_\nu g_{\lambda\beta})$

Trabajamos la exprecion $g^{\beta\mu} \partial \Gamma^{\lambda}_{\lambda\beta}$

$= \frac{1}{2} g^{\beta\mu} g^{\lambda\nu} (\partial_\beta dg_{\nu\lambda} + \partial_\lambda dg_{\nu\beta} - \partial_\nu dg_{\lambda\beta})$

β, ν, λ son indices mudos, entonces intercambiamos β por ν, ν por β y λ por α

$= \frac{1}{2} g^{\nu\mu} g^{\alpha\beta} (\partial_\nu dg_{\beta\alpha} + \partial_\alpha dg_{\nu\beta} - \partial_\beta dg_{\alpha\nu})$

$J^{\mu} = K \partial_\beta dg_{\nu\alpha} + K \partial_\alpha dg_{\nu\beta} - K \partial_\nu dg_{\alpha\beta} - K \partial_\nu dg_\beta \alpha - K \partial_\alpha dg_{\nu\beta} + K \partial_\beta dg_{\alpha\nu}$

$J^{\mu} = 2K (\partial_\beta dg_{\nu\alpha} - \partial_\nu dg_{\alpha\beta}) = g^{\mu\nu} g^{\alpha\beta} (\partial_\beta dg_{\nu\alpha} - \partial_\nu dg_{\alpha\beta})$

$\boldsymbol{= g^{\mu\nu} g^{\alpha\beta} (\partial_\alpha dg_{\nu\beta} - \partial_\nu dg_{\alpha\beta})}$